Importance of Fractal Structures in Modern Biology and Discussion of Forward Kolmogorov Equation With Fractional Derivative Approach

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Euclidean geometry does not completely describe nature at the present time. Countless physical and biological phenomenon in nature such as clouds, coastlines, mountain formations, molecular structures of crystals, plants can be described more accurately by fractal geometry. Fractal structures, basically formed by repetition and show features such as full self-similarity, semi-self-similarity, statistical self-similarity and complex geometric objects with fractional dimensions. Mathematically, a fractal can be defined as a set in which the Hausdorff dimension exactly exceeds the topological dimension. Living systems exhibit fractal characteristics from micro-scale to organism size and even to collective lives. Therefore, we can say that modern biology research is directed towards fractal structures in the modeling of biological processes and beings at both molecular and system biology levels. However, the importance of statistical understanding in the formalization of natural laws increases the interest of natural scientists in statistical and stochastic processes. For example, diffusion is a very important process for biology. On the other hand the normal distribution models of diffusion are not universal. Biological phenomena can be modeled more realistically with "abnormal convection" events that do not fit the Gaussian distribution, where fractal properties are also important. One of the mathematical tools used in modeling diffusion is the Fokker-Planck equation, also known as the forward Kolmogorov equation. The Fokker-Planck equation is a partial differential equation which gives time evolution of the probability density of a system under the influence of stochastic behavior. In this study, firstly the basic properties of fractal structures, their formation by random processes and their current use in biology will be discussed. We propose to use the fractional Fokker-Planck equation, which we have written using the fractional derivative approach, in the modeling of systems with fractal structure and compare the solutions of the fractional Fokker-Planck equation with the solutions of the classical Fokker-Planck equation. We fundamentally argue that the use of fractional equations in simulations of natural phenomena is not a mathematical exercise but also a more realistic approach to understanding deviations from normality.